Also, we know from the theory of the dynamic absorber [2] that its efficiency depends nonmonotonically on the damping.

Figure 5 shows the function T(k) for M = 0.2, ε = 0.2, a = 5, ℓ = 4.5, and c = 0.05, 2, 3, and 4 (curves 1-4); this figure exhibits the major influence of the position of the resonator in the region Ω on the absorption of acoustic vibrations. The mechanism of this influence can be identified with the displacement of the field of acoustic disturbances radiated from the resonator relative to the normal mode of acoustic vibrations in the main region.

In conclusion the author is grateful to V. A. Yudin for writing the program for the calculations.

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INITIATION OF COHERENT MOTION IN TURBULENT COCURRENT FLOWS

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UDC 532.517.4+532.526

It has been established experimentally that organized motion is present in all turbulent shear flows. The presence of coherent motion in flows served as the basis for Townsend's creation [1, 2] of a turbulence model with a binary structure in which the undisturbed surrounding fluid is brought into the shear flow by coarse eddies which develop against a background of small-scale turbulence. Townsend also developed the hypothesis of the universal similitude of free shear flows. In accordance with this hypothesis, at a sufficiently great distance from the source, motion is determined by the local scales of velocity and length. The scales depend on the type of flow and the external velocity and length scales. The average motion, referred to the local scales, is described by universal functions which depend only on the method by which the motion comes about. Coarse eddies are in dynamic equilibrium with the average flow. This subsidiary condition determines the form and intensity of these eddies. Similitude has been proven to exist for plane shear layers [3-6], plane wakes [7-9], axisymmetric wakes [10-14], and axisymmetric shear layers and plane jets [15]. Here, characteristic local values of velocity and length are used as the scales. However, such scales depend to a significant extent on the experimental conditions (the presence of small harmonic perturbations [4-6, 8] and external turbulence [16] and, for cocurrent flows, the form of the body [7-14]) and other features of the experiment. The type of load and its characteristic frequency and scale are reflected in the coherent structures present in these flows. Some authors [6, 8] have attempted to describe external effects by using the theory of hydrodynamic stability of inviscid flows. This theory can be used to analyze the response of a small harmonic perturbation.

The memory of the initial conditions by the flow is a generally recognized factor as well, at least for the ranges which have been studied. However, it is not yet clear whether or not universal asymptotic similitude exists for each type of free shear flow. It is difficult to explain the absence of such similitude in turbulent flows as being the result of intensive energy transfer between different scales of motion. Coherent large-scale structures have been recorded in developed turbulent flows at very large distances from the source. The mechanism of their reproduction may be hydrodynamic instability of the average flow. If a turbulent shear flow is modeled as a flow with a certain effective viscosity ν_t, then the corresponding turbulent Reynolds numbers (Re_t) will be finite and will determine whether the flow will be stable or unstable against longwave perturbations. At values of Re_t less than the critical value, the flow will be stable, and degeneration of small-scale turbulence will result in a decrease in ν_t and a consequent increase in Re_t. The flow will

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then become unstable and the large-scale motion will draw energy from the average flow and transmit it to smaller scales, thus increasing ν_t. The critical Re_t will determine the limiting similarity regime toward which the given flow will strive.

It will be shown below that Re_t is an integral parameter which communicates information on the initial conditions to the local scales. We find the critical value of Re_t for the flow in an axisymmetric far wake. We will also examine the flow behind a self-propelled body.

1. Self-Similar Turbulent Wakes. We will refer to a wake as a change in the velocity U_∞ of a uniform flow incident on a stationary body. The motion of a continuum which occurs with passage by a body at a constant velocity U_∞ is usually referred to as a cocurrent flow. The Boussinesq hypothesis on eddy viscosity has been used with success in the phenomenological theory of turbulent free shear flows [1, 17]. The Reynolds stresses are modeled as viscous stresses with an effective viscosity ν_t which is constant in each section. According to [1], an expedient method of determining the mean value of ν_t is to compare the observed distribution of mean velocity with the value calculated from this model. A similarity solution is obtained by using the hypothesis of the similitude of turbulent pulsations resulting in a transfer of momentum. Thus, ν_t can be represented in the form $\nu_t(X) \sim bu_0$, where $u_0(X)$ and $b(X)$ are local velocity and length scales; X is the coordinate directed along the flow. The Reynolds number constructed from this viscosity and the local scales is constant over the entire region in which the flow is self-similar:

$$\mathrm{Re}_t = u_0 b/\nu_t (\equiv \mathrm{const}). \tag{1.1}$$

Since the eddying fluid transported by a turbulent flow is attributed the properties of an actual continuum but with a higher viscosity, the transverse dimension of the wake will grow as a result of the diffusion of vorticity at a rate determined by ν_t:

$$b = [\nu_t (X - X_0)/U_\infty]^{1/2} \tag{1.2}$$

(X_0 is the hypothetical beginning of the self-similar wake). One more condition is needed in order to find u_0, b, and ν_t. For impulsive, planar, and axisymmetric wakes, this is the conservation-of-momentum condition

$$u_0 b = \mathrm{const},\ u_0 b^2 = \mathrm{const}. \tag{1.3}$$

Equations (1.1)-(1.3) lead to the well-known laws for the self-similar development of impulsive wakes

$$u_0 \sim (X - X_0)^{-1/2},\quad b \sim (X - X_0)^{1/2},\quad \nu_t = \mathrm{const};$$
$$u_0 \sim (X - X_0)^{-2/3},\quad b \sim (X - X_0)^{1/3},\quad \nu_t \sim (X - X_0)^{-1/3}.$$

For non-impulsive wakes, the conservation integral was obtained in [18] with the condition ν_t = const

$$u_0 b^3 = \mathrm{const},\ u_0 b^4 = \mathrm{const}, \tag{1.4}$$

This in turn leads us to an expression for the local scales of non-impulsive wakes

$$u_0 \sim (X - X_0)^{-3/4},\quad b \sim (X - X_0)^{1/4},\quad \nu_t \sim (X - X_0)^{-1/2};$$
$$u_0 \sim (X - X_0)^{-4/5},\quad b \sim (X - X_0)^{1/5},\quad \nu_t \sim (X - X_0)^{-3/5}.$$

We represent the mean flow velocity in self-similar far wakes in the form

$$U = U_\infty [1 - \varepsilon\varphi_0(r)],\ V = \varepsilon^2 U_\infty \chi_0(r) \tag{1.5}$$

[$\varepsilon = u_0/U_\infty (\ll 1)$, $r = y/b$ is the dimensionless transverse coordinate]. By inserting (1.5) into the averaged equations of motion and limiting ourselves to terms on the order of ε, we obtain the following for axisymmetric flows

$$\varphi_0 \frac{d\varepsilon}{dX} - \varepsilon \frac{d\ln b}{dX} r \frac{\partial\varphi_0}{\partial r} = \frac{\varepsilon}{X - X_0} \frac{\partial}{r\,\partial r}\left(r \frac{\partial\varphi_0}{\partial r}\right). \tag{1.6}$$

Here, we made use of Eq. (1.2). We use the continuity equation to find the following for the radial component of mean velocity

$$\frac{d\ln b}{dX} r \frac{\partial\varphi_0}{\partial r} - \frac{d\ln\varepsilon}{dX}\varphi_0 = \frac{\varepsilon}{b}\frac{\partial(r\chi_0)}{r\,\partial r}. \tag{1.7}$$

In the case of an impulsive wake, the solution of Eqs. (1.6) and (1.7) has the form

$$\varphi_0 = \exp(-ar^2),\ \chi_0 = r\varphi_0(r)/(3\mathrm{Re}_t), \tag{1.8}$$

It is usually the case that a = ln 2. Thus, as the transverse scale b we take distance along the flow axis over which the deficit of mean velocity is equal to half its value on the axis. We introduce the notation r_0 for this scale. Proceeding as before and limiting ourselves to terms of the order ε, we write an equation which follows from the law of conservation of momentum J and connects the local scales r_0, u_0 with Eqs. (1.5), (1.8) for mean velocity:

$$r_0^2 u_0 = \frac{\ln 2}{\pi} \frac{J}{\rho U_\infty}. \tag{1.9}$$

Using (1.1) and (1.2) in combination with (1.9) [with the transverse scale in (1.1)-(1.2) being defined as r_0], we obtain the following for the local scales of a self-similar impulsive wake

$$r_0 = C[(X - X_0)/\mathrm{Re}_t]^{1/3},\ u_0/U_\infty = C\,[(X - X_0)/\mathrm{Re}_t]^{-2/3}, \tag{1.10}$$

where the constant $C = [(J \ln 2)/(\pi\rho U_\infty^2)]^{1/3}$ depends only on the drag of the body. The quantity Re_t in (1.10) characterizes individual features of flows behind bodies with identical form drag; Re_t can be found experimentally by means of (1.10):

$$\mathrm{Re}_t = (X - X_0)u_0/(r_0 U_\infty). \tag{1.11}$$

The inverse Re_t^{-1} coincides with the quantity S_0 introduced in [14]. We used local scales of type (1.10), which allowed us to generalize a great deal of empirical data on the mean characteristics of axisymmetric impulsive wakes. The expressions found experimentally for the local scales differ from (1.10) by $[\pi/(4 \ln 2)]^{1/3}$, i.e., by about 4%. To illustrate, we will present several values of Re_t calculated with (1.11) from published experimental data: a disk [10], Re_t = 0.5; a sphere [13], 0.8; an ellipsoid [11], 6.5; a porous disk [13], 7.0. The increase in Re_t reflects the level of the Reynolds stresses in the wakes behind the bodies taken as examples.

In the case of a non-impulsive wake, we write the solution of Eqs. (1.6) and (1.7) in the form

$$\varphi_0 = (1 - kr^2)\ \exp(-kr^2),\ \ \chi_0 = (r/5)(2 - kr^2)\exp(-kr^2). \tag{1.12}$$

As the transverse scale b, we take the distance over which the velocity in the axial jet is equal to half its maximum value. This corresponds to k = 0.31492. We retain r_0 for this scale. Having designated the constant in (1.4) as N, we rewrite the local scales of the non-impulsive axisymmetric wake:

$$r_0 = C_1[(X - X_0)/\mathrm{Re}_t]^{1/5},\ u_0/U_\infty = C_1[(X - X_0)/\mathrm{Re}_t]^{-4/5},\quad C_1 = (N/U_\infty)^{1/5}. \tag{1.13}$$

The value of Re_t is determined by Eq. (1.11). Several variants of a non-impulsive axisymmetric wake were realized in [14, 19-22]. It is difficult to obtain a completely non-impulsive flow experimentally. At a certain distance from the source, an imbalance of the momentum flux leads to the formation of a flow of type (1.10). Nevertheless, Eqs. (1.13) are valid in an intermediate region where the character of the flow is of a distinctly non-impulsive nature (see [14]).

Introduction of the quantity Re_t made it possible to generalize experimental data on the average flow. However, the fluctuation characteristics of turbulent flow in wakes cannot be described in the same similarity variables. In keeping with the hypothesis advanced as the basis of the present study, the dynamics of a wake is significantly influenced by large-scale perturbations. Due to hydrodynamic instability, the amplitude of these perturbations changes downstream in accordance with a law which differs from the similarity law characteristic of the average flow. The exact law of change in the amplitude and the form of the perturbations depends on their scale and Re_t, as will be shown below.

2. Formulation of the Problem of the Stability of Turbulent Wakes. The response of turbulent flows to a large-scale external wave disturbance can be studied on the basis of the linear theory of hydrodynamic stability of viscous flows [23]. The velocity field in the cylindrical coordinate system (X, R, φ) has the components (u, v, w). According to [23], the velocity and pressure fields are represented in the form $\mathbf{u} = \mathbf{U} + \mathbf{u}' + \tilde{\mathbf{u}}$. Along with the average flow $\mathbf{U}$ and turbulent pulsative motion $\mathbf{u}'$, this expression contains the regular wave motion $\tilde{\mathbf{u}}$. The distribution of the mean velocity of the two types of flows being examined is represented by Eqs. (1.5), (1.8), and (1.12). Figure 1 shows the longitudinal components of the mean velocities of these flows. The equations of motion and continuity for the wave motion are written as

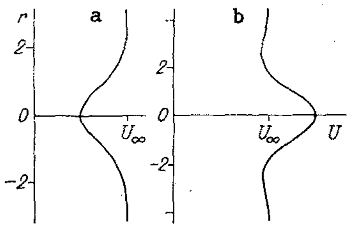

Fig. 1

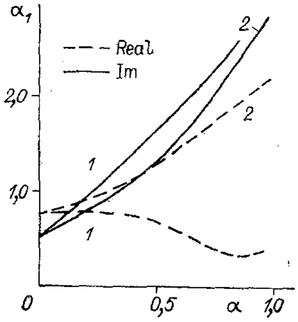

Fig. 2

$$\frac{D\tilde{u}_i}{Dt} + (\tilde{u}_j\nabla_j)U_i = -\frac{1}{\rho}\nabla_i\tilde{p} + \nabla_j(2\nu_t\tilde{e}_{ij}), \quad \nabla_j\tilde{u}_j = 0 \tag{2.1}$$

($\tilde{e}_{ij}$ is the strain-rate tensor associated with the wave motion). We ignore molecular viscosity compared to eddy viscosity ν_t. By virtue of the presumed similarity of the average flow, we will also seek the perturbations in similarity form

$$\begin{Bmatrix} (\tilde{u}, \tilde{v}, \tilde{w}) \\ \tilde{p} \end{Bmatrix} = \begin{Bmatrix} \varepsilon U_\infty [u(r), v(r), w(r)] \\ \varepsilon^2 U_\infty^2 q(r)/\mathrm{Re}_t \end{Bmatrix} \exp(i\Theta), \tag{2.2}$$

where $\partial\Theta/\partial X = \alpha^0 + (\varepsilon/\mathrm{Re}_t)^{\cdot}\alpha_1^0(X)$; $\partial\Theta/\partial\varphi = m$; $\partial\Theta/\partial t = -\omega^0$; $r = R/r_0$; ω^0 is the angular frequency of the linear oscillations; m is the azimuthal wave number; $\alpha^0 = \omega^0/U_\infty$ is the longitudinal wave number of perturbations propagating with the phase velocity U_∞. The last relation is valid at $\varepsilon \to 0$, since the change in phase velocity cannot be greater than the deficit of external velocity. The second term, containing the complex number α_1^0, represents the correction for motion in the wake. The real part of α_1^0 reflects the drift of the phase of the perturbations in a coordinate system moving with the velocity U_∞, i.e., in the wake. The imaginary part of α_1^0 is connected with the change in the amplitude of the perturbations, which characterizes their stability or instability (depending on the sign). After we insert (2.2) into (2.1) and change over to the new variables (Θ, X, r, φ), we have a system of ordinary differential equations with coefficients dependent on the coordinate X. It was found in experimental studies that the wavelength of the most dangerous large-scale perturbations changes in proportion to the local length scale [24]. This means that

$$(\alpha^0, \alpha_1^0) = (\alpha, \alpha_1)/r_0, \quad (\alpha, \alpha_1) = \mathrm{const}. \tag{2.3}$$

For far wakes, $\varepsilon \ll 1$. Thus, we will limit ourselves to terms on the order of ε in the resulting system. With allowance for (2.3) and the relation $\alpha^0 = \omega^0/U_\infty$, the terms on the order of $O(\varepsilon^2)$ form the system of equations

$$\begin{gathered} \beta^2 u - \tau r u' + i\alpha q - \mathrm{Re}_t\varphi_0' v - (ru')'/r = 0, \\ \beta^2 v - \tau r v' + q' + v/r^2 + i2mw/r^2 - (rv')'/r = 0, \\ \beta^2 w - \tau r w' + imq/r + w/r^2 - i2mw/r^2 - (rw')'/r = 0, \\ i\alpha u + (rv)'/r + imw/r = 0, \ \beta^2 = i(\alpha_1 - \alpha\mathrm{Re}_t\varphi_0) + \alpha^2 + m^2/r^2 - (1-\tau)\mu \end{gathered} \tag{2.4}$$

($\tau = 1/3$ for an impulsive wake and $\tau = 1/5$ for a non-impulsive wake, $\mu = 1$). If we introduce $U_S = -\varphi_0$, $c = -\alpha_1/(\alpha\mathrm{Re}_t)$, then at $\tau = 0$, $\mu = 0$ system (2.4) takes the form examined in [25]. The prime denotes a derivative with respect to r. The boundary conditions for the perturbation are given by the relations

$$\begin{gathered} u,\ v,\ w,\ q \to 0 \ \text{at} \ r \to \infty, \\ u(0) = q(0) = 0,\ m \neq 0,\ v(0) = w(0) = 0,\ m \neq 1,\ v(0) + iw(0) = 0, \\ m = 1. \end{gathered} \tag{2.5}$$

The solution of the problem of the stability of a flow consists of finding eigenvalues α_1 and corresponding eigenfunctions (u, v, w, q) of boundary-value problem (2.4), (2.5). The eigenvalue problem was solved numerically by the differential trial-run method, with joining of the different parts of the solution in the critical layer [26]. Certain difficulties are encountered when attempting to solve the equations for the correction factors, due to the presence of a singularity on the axis. Equations (2.4) have a regular singularity at r = 0. The solution of the system can be obtained in the neighborhood of this singularity in the form of a series in powers of r:

$$(v,\ w) = r^{\gamma}(a_i + b_i r^2 + \ldots),\ i = 1,\ 2,$$
$$(u,\ q) = r^{\gamma-1}(a_i + b_i r^2 + \ldots),\ i = 3,\ 4. \quad (2.6)$$

Inserting (2.6) itno (2.4) and collecting the terms with identical powers of r, we obtain the characteristic equation for γ and recursion formulas that link the constants in expansion (2.6). The roots of the characteristic equation are equal to (m + 1), (1 − m), −(m + 1), (m − 1), with the first two being quadratic. We will write out three linearly independent solutions for m ≠ 0 which are finite at r = 0:

$$\{-r^m,\ [i\alpha/(m+1)/2]r^{m+1},\ [\alpha/(m+1)/2]r^{m+1},\ 0\},$$
$$\{0,\ [m/(m+1)/4]r^{m+1},\ [i(m+2)/(m+1)/4]r^{m+1},\ r^m\}, \quad (2.7)$$
$$\{0,\ r^{m-1},\ ir^{m-1},\ 0\}.$$

Expressions (2.7) are used to find the correction matrix and its first derivative at r = 0. These values are needed to solve the equations for the correction factors. The order of system (2.4) may be reduced to four in the case m = 0. Using the form of the boundary conditions for r = 0, it is not hard to write two linearly independent solutions in the neighborhood of the axis. In numerical calculations, the condition of decay of the perturbations at infinity was replaced by the condition of attachment at a certain sufficiently great distance from the axis R_0. Small α requires an increase in R_0, since longwave perturbations are very sensitive to the conditions on the external boundary. This is illustrated in Fig. 4, where the triangles show the calculation of the neutral curve with $R_0 = 12$. To keep the boundary conditions from affecting the results, the interval of integration was changed in accordance with the law $R_0 = c_0/\alpha$. It was found that at $c_0 = 8$ a further increase in R_0 with fixed α has no effect on the results of calculation of the eigenvalues. The equations for the correction factors were solved by the Runge–Kutta method with a constant step. We doubled the integration step in the neighborhood of the critical layer to improve the accuracy of the calculations. The numerical algorithm was checked against the data in [25].

3. Stability of an Impulsive Axisymmetric Wake. The azimuthal wave number changes discretely and is represented by a denumerable set of integers m = 0, 1, 2,.... The spectrum for each m is also denumerable. Of particular interest are the spectral modes corresponding to the most dangerous perturbations for Re_t from the range established experimentally with flows past bodies of different shapes (Re_t[0.5-7.0], see above). The complexity of the disturbed motion increases with an increase in the azimuthal wave number. As a rule, more complex motion dies out more quickly. Thus, we examined only m = 0, 1, 2. The principle underlying the approach being taken here allows us to find the most dangerous modes for fixed m, α, and Re_t. Curves 1 and 2 in Fig. 2 show eigenvalues for axisymmetric perturbations (m = 0) in relation to α. These values correspond to $Re_t = 2.0$ and 5.0. At α = 0, the eigenvalues of boundary-value problem (2.4), (2.5) are independent of Re_t. Figure 3 shows eigenfunctions of axisymmetric perturbations at $Re_t = 5.0$ and α = 0.5. The axial component of the velocity of the perturbed motion has a maximum on the axis of the wake. The spiral perturbations (m = 1) are the most dangerous throughout the range of Re_t. Whereas perturbations with m = 0, 2 are absolutely stable, spiral perturbations have a region of parameter values for which such motion is unstable. The regions of the existence of stable and unstable perturbations with m = 1 are shown in Fig. 4. The critical values of the parameters of the neutral perturbations $Re_{t*} = 6.78$, $\alpha_* = 0.098$, $\alpha_{1r} = 0.079$. In accordance with the inviscid analysis [25], $\alpha \to 0.99$ at $Re_t \to \infty$. Calculations performed in [25] for a viscous axisymmetric wake with a similar velocity deficit yielded the critical values of the parameters of neutral perturbations $Re_* = 23.05$, $\alpha_* = 0.41$, $\alpha_{1r} = 2.93$. These studies were conducted with the assumption that the initial flow was parallel. As has been shown by our calculations, allowance for the effects connected with nonparallelism of the flow in the wake leads to appreciable expansion of the region of parameters for which the given flow is unstable. Figure 5 shows eigenvalues for spiral perturbations in relation to α for $Re_t = 1.0$ and 5.0 (curves 1 and 2), these values corresponding to the most dangerous spectral mode. With a decrease in Re_t, the eigenvalues are given by the asymptotic expression $\alpha_1 = i\alpha^2$. Figure 6 shows the distributions of pressure and velocity in the case of spiral perturbations for $Re_t = 5.0$ and α = 0.5. The maximum of axial velocity is located a certain distance from the axis of the wake. The sign of the helicity of the perturbations was obtained in every experiment from studying axisymmetric turbulent wakes (see [13], for example). The maximum degenerates as the wake develops, which is typical of the decay of spiral disturbances. This fact is consistent with the results of the present study, since $Re_t \lesssim Re_{t*}$ for most of the empirically investigated axisymmetric wakes. The generation and development of large-scale

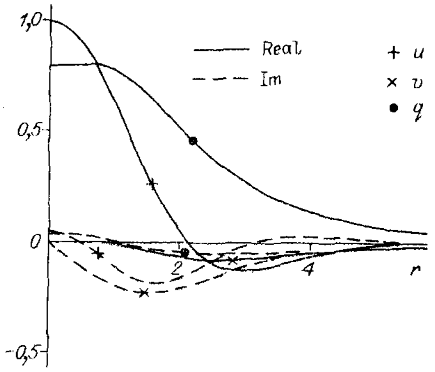

Fig. 3

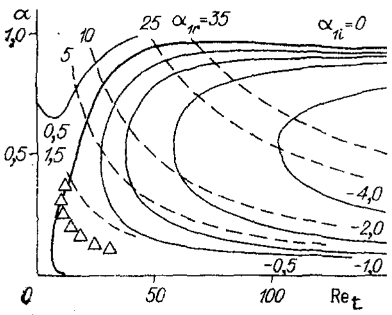

Fig. 4

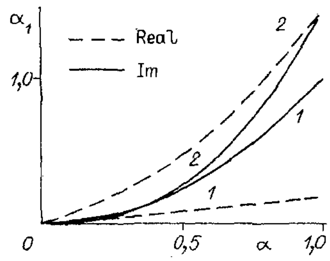

Fig. 5

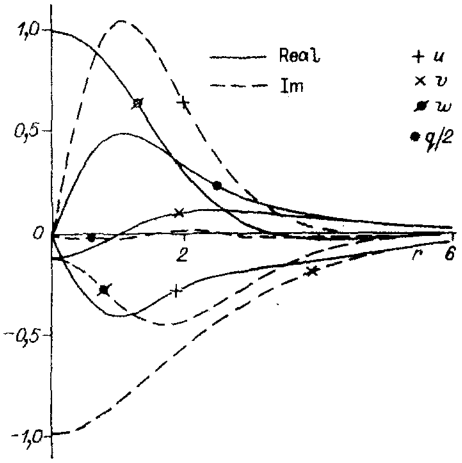

Fig. 6

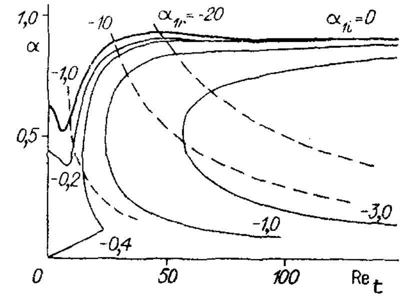

Fig. 7

spiral instabilities was observed in the visualization of a wake created by pulling a rope through the flow [14]. At small Re_t, an increase in the azimuthal wave number greatly complicates the structure of the spectrum. For m = 2 and $Re_t \to 0$, the least stable spectral mode, having an inviscid asymptote, corresponds to the expression $\alpha_1 = i(\alpha^2 + 1.72)$.

It follows from the definition of ε and Eqs. (1.11), (2.2), and (2.3) that the factor connected with a nonsimilar change in the amplitude of the perturbations has the form

$$(u', v', w') \sim \varepsilon U_\infty (X - X_0)^{i\alpha_1}. \tag{3.1}$$

In accordance with the above analysis, small values are obtained for the decrements of certain longwave perturbations for values of Re_t from the range found experimentally in the case of flow past bodies of different shapes. This fact, together with (3.1), confirms the very slow growth of perturbations in impulsive axisymmetric wakes.

4. Stability of a Non-Impulsive Wake. Figure 7 shows regions of existence of stable and unstable spiral perturbations of the given flow in the plane (Re_t, α). Due to the presence of the point of inflection and the effects of nonparallelism, the flow in the non-impulsive turbulent wake is absolutely unstable against longwave perturbations with m = 1. At $Re_t \to 0$, the eigenvalues are given by the asymptotic expression $\alpha_1 = i(\alpha^2 - 0.4)$. The increment retains the constant value $\alpha_1 = -i0.4$ along the x axis. As for the impulsive wake, perturbations with m = 0, 2 are absolutely stable. The distributions of the pressure and velocity of the perturbations are on the whole similar to those obtained for an impulsive wake. As was already noted, it was very complicated to study this type of flow experimentally due to the difficulty of exactly satisfying the condition of non-impulsiveness of the wake. The lack of reliable data makes it impossible to compare experimental and theoretical results on the development of perturbations in flows of this type.

5. Conclusions. The Boussinesq hypothesis on eddy viscosity and the hypothesis on the similarity of turbulent pulsations that exchange momentum make it possible to find the form of the local velocity and length scales. These scales contain an empirical parameter which determines their dependence on the initial conditions of formation of a wake. This parameter

is Re_t. A cascade of large-scale instabilities can serve as a mechanism for the loss of information on initial conditions by a flow. At the same time, it is a well-established empirical fact that, for the distances that have been investigated, nearly all free shear flows have a "memory" of the initial conditions. Large coherent structures formed at the beginning of the flow change little in shape with motion downstream and are seen at great distances from the source. This conservatism of coherent structure can be attributed to their weak interaction with the average flow. In order to determine the response of a flow to a small external disturbance, we performed a linear analysis of the stability of axisymmetric turbulent wakes. The analysis showed that the amplitude of the perturbations changes in accordance with a power law. Along with the corresponding similarity part, the exponent contains a number which characterizes the growth or decay of the perturbations. It depends on Re_t and has an absolute value on the order of or less than unity for the impulsive axisymmetric wakes investigated here. In accordance with this result, it can be concluded that the "memory" for the initial conditions is due to the weak interaction of large-scale perturbations with the average flow. The time of formation of universal similarity in cocurrent flows is determined by the characteristic time of development of large-scale perturbations.

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EVOLUTION AND INTERACTION OF THREE-DIMENSIONAL VORTEX CLUSTERS

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TURBULENT FLOW MODEL: ENSEMBLE OF SMALL VORTICES

Hydrodynamic instabilities in turbulent flows lead to the formation of concentrated pockets of vorticity (clusters). Their evolution in time is governed by the nonlinear vorticity dynamics in the interior of the vortices and by their mutual interaction.

The possibility of analyzing separately the internal and external degrees of freedom depends on the intermittency factor $\varkappa = \lambda/\ell$ (λ is a characteristic length of the vortices, and ℓ is the distance between them). If $\varkappa \to 0$, the vortices interact only through their momenta, and the other degrees of freedom are insignificant [1, 2].

If $\varkappa \neq 0$, other multipole moments take part in the interaction of the vortices. In turn, their evolution is determined not only by the effect of the surroundings on each specific vortex, but also by the nonlinear dynamics of all internal degrees of freedom, the set of which is not exhausted by the multipole moments [2, 3].

The influence of the vortex surroundings on its internal degrees of freedom for $\varkappa \ll 1$ is similar to the influence of a certain nonuniform external velocity field. Consequently, the total system of equations for the ensemble of small vortex clusters is partitioned into subsystems. Each subsystem describes a particular vortex in the external field induced by the other vortices. The objective of the present study is to derive such a subsystem of equations and to analyze its solutions.

VORTEX CLUSTER IN AN EXTERNAL FIELD IN AN INFINITE COMPRESSIBLE FLUID

The vorticity field obeys the equation

$$\partial\boldsymbol{\omega}/\partial t - \nu\Delta\boldsymbol{\omega} = (\boldsymbol{\omega}\nabla)\mathbf{u} - (\mathbf{u}\nabla)\boldsymbol{\omega}. \tag{1}$$

Galerkin's method is used for the approximate solution of Eq. (1). The choice of basis for the expansion is based on the following considerations.

Vortex clusters in turbulent flows comprise certain irregular diffuse formations. If the Reynolds number Re determined from the cluster parameters is small, the evolution of a vortex depends mainly on the viscosity. Consequently, a natural basis for the expansion is the set of solutions of the linearized equation (1).

Turbulent fluctuations having a broad spectrum of space scales develop inside the vortices for large Re. The detailed description of these fluctuations would require the inclusion of a large number of terms in the expansion, regardless of the system of functions chosen as the basis. Large-scale vortex deformations, which influence the interaction between the vortices, are the most important in regard to the present study. Small-scale fluctuations act as a reservoir, from which energy is drained. Their influence can be taken into account by means of an effective viscosity coefficient ν_{ef}. The number Re_{ef} formulated using the effective viscostiy is no longer as large as Re, and the solution of the linearized equation (1) with ν replaced by ν_{ef} can be adopted as the basis of the expansion.

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